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COLLECTIVE ION ACCELERATION AT VERY HIGH ENERGY IN A STATIC MAGNETIC FIELD*

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A method for accelerating ions is described in which extremely relativistic electrons are injected in a static magnetic field and form a ring which subsequently is compressed by a sequence of axial accelerations and decelerations. Ions trapped in the electrostatic field of the electron ring can in principle be accelerated in a decreasing magnetic field to very high energy (γ_i) where $\gamma_i < \gamma_e$ and γ_i, γ_e are the final ion and electron energy, respectively, expressed in rest mass units. The electrons in the ring are confined by an external magnetic field. Radial stability of the trapped ions is provided by imposing on them an azimuthal momentum equal and opposite to the electron azimuthal momentum so that both ions and electrons are magnetically focused by the external magnetic field. The ions are initially accelerated by adiabatic expansion of the ring in a decreasing magnetic field; subsequently the ring is decelerated at constant radius. During this step the ions acquire an azimuthal mechanical momentum almost equal to that of the relativistic electrons. The azimuthal momentums of the ions and electrons are subsequently equalized by allowing the electrons to lose energy by synchrotron radiation. Preliminary parameters are presented for such a static-magnetic-field ion accelerator capable of producing 1000-GeV ions.

I. Introduction

A method is discussed in this report to accelerate ions trapped in a ring of extremely relativistic electrons to very high energies, of the order of 1000 GeV, by axial acceleration of the electron ring, at constant radius, in a decreasing magnetic field. It is obvious that the electron energy γ_e (expressed in rest mass units, rmu) should be larger than the ion energy γ_i . Since it is desired to accelerate the ions up to $\gamma_i \approx 1000$ it follows that the initial electron energy should be of the order of $\gamma_e \approx 2000$ or 1 GeV. In a companion report¹ a method is described where the electron ring alternately "rolls" and "coasts," undergoing axial acceleration at a constant radius followed by deceleration and adiabatic compression in a decreasing magnetic field. In the process the radius of the ring shrinks and its electrostatic field increases as the compression ratio to the 3/2 power. During the acceleration of the ions the alternation of rolling and coasting of the ring is repeated but during the adiabatic compression phase the ring is not allowed to regain all its azimuthal momentum. Thus the ions are accelerated to an energy $\gamma = \gamma_i$, then decelerated to $\gamma_i/2$. In the next cycle they are accelerated again to an energy $\gamma = \gamma_i/2$ and so on. For example, if during the first acceleration at con-

stant radius the ion energy achieved is 4 rmu, the minor radius of the ring expands by a factor of two. In the subsequent adiabatic compression the ring is compressed by a factor of two and the minor radius is restored to its value at the beginning of the ion acceleration. At the same time the current in the ring is doubled while the ion energy is 2 rmu. As the process continues the accelerating field (i.e., the electrostatic field of the ring) increases linearly with the ion energy until a limit radius is achieved, a few centimeters, below which it is not technologically possible to build the inside coils required for the acceleration at constant radius. It appears that with this process fields as high as 100 MeV/cm can be achieved, making possible acceleration of the ions to an energy of 1000 GeV with an acceleration length of the order of 1000 m.

The radial force exerted on the ions by the field (E) of the electron ring is proportional to E/γ_i . Although at the beginning of the acceleration the electric field E is comparable to the external field, the radial forces are diminishing very fast as the ion energy γ_i increases. Consequently in order to confine the ions within the ring it is necessary to impart to them an azimuthal mechanical momentum equal and opposite to the azimuthal momentum of the electrons. In this way the ions are confined magnetically as the electrons are, and both are collocated at the same equilibrium orbit. As a result both the electrons and the ions undergo the same change in azimuthal mechanical momentum during the acceleration at constant radius or deceleration in the adiabatic compression phases. Thus the acceleration of the ions is divided into two steps: first azimuthal acceleration to provide them with an azimuthal mechanical momentum equal to that of the electrons, then axial acceleration to give them very high energy.

II. Azimuthal Acceleration of the Ions

Preceding this step is the phase of compressing the ring to a small radius to enhance its electrostatic field. In a companion report¹ a ring of 1000 MeV, 1000 A is followed during its compression phase in a static magnetic field. The final parameters of the ring before the trapping of the ions are:

Electron energy	$\gamma_e = 2000$ rmu
Ring current	$I_e = 29,400$ A
Major radius	$r_0 = 34$ cm
Minor radius	$a_0 = 0.092$ cm
Loading factor*	$\xi = 0.64$
External field	$B_0 = 100,000$ G
Electrostatic field of the ring	$E = 19.2$ MeV/cm

*Work performed under the auspices of the U. S. Atomic Energy Commission.

*Loading factor is the ratio of the self-field at zero axial velocity to the external field B_0 .

After trapping of the ions ($N_i \approx 10^{-4} N_e$) the process followed in the static compressor is reversed. The ring is adiabatically expanded, and the ions are accelerated to an energy

$$\gamma_{i0} = \frac{r_1}{r_0}. \quad (\text{II.1})$$

The azimuthal momentums of the electrons and ions are respectively

$$p_{\theta e} = (\gamma mc) \frac{r_0}{r_1}, \quad (\text{II.2a})$$

$$p_{\theta i} = 0. \quad (\text{II.2b})$$

The adiabatic expansion is followed by deceleration at constant radius until the ring practically stops. The electrons regain all their azimuthal momentum,

$$p_{\theta e} = \gamma mc, \quad (\text{II.3a})$$

while the momentum of the ions is

$$p_{\theta i} = \left(1 - \frac{r_0}{r_1}\right) \gamma mc. \quad (\text{II.3b})$$

Also we have

$$p_{ze} = p_{zi} = 0. \quad (\text{II.3c})$$

The electrons should now undergo an azimuthal deceleration in order to lose $(r_1/r_0)\gamma mc$ of azimuthal momentum, thus achieving equal momentum for the ions and electrons. Therefore, we need a process which decelerates the electrons but not the ions. Nature provided such a process in the form of incoherent synchrotron radiation loss, which is

$$\dot{\gamma} = 2 \times 10^{-15} B^2 \gamma^2 \text{ rmu}/\mu\text{s}. \quad (\text{II.4})$$

The time required for an energy loss $(\gamma_1 - \gamma_2)mc^2$ is

$$\tau = 50 \left(\frac{10^3}{\gamma_1} \right) \left(\frac{B}{10^5} \right)^2 \left(\frac{\gamma_1 - \gamma_2}{\gamma_2} \right) \mu\text{s}. \quad (\text{II.5})$$

But

$$\gamma_1 - \gamma_2 = \frac{r_0}{r_1} \gamma_1. \quad (\text{II.6})$$

Then

$$\tau = \frac{50}{\frac{r_1}{r_0} - 1} \frac{10^3}{\gamma_1} \left(\frac{B}{10^5} \right)^2 \mu\text{s}. \quad (\text{II.7})$$

According to the numerical example for $r_1 = 5r_0$, $\gamma_1 = 2000$, $B = 0.2B_0$ we find

$$\tau = 125 \mu\text{s}.$$

This is a rather long time in view of the fact that all the other processes in the acceleration from the injection of the electrons to the acceleration of the ions last less than $4 \mu\text{s}$. For example, coils located just outside the electron ring can provide very strong alternating fields, up to 200 kG (which can be pulsed for a few microseconds), in the form

$$B = B_z \cos(P\theta) \quad (\text{II.8})$$

where P is a large integer. If this configuration is not otherwise disturbing to the ring it will result in reduction of the time required for the electron cooling by synchrotron radiation to approximately $3 \mu\text{s}$.

During the ring compression phase all the internal coils can be cantilevered from the injection end of the accelerator. The total length of the static compressor is approximately 50 m. The length (z) of the adiabatic expansion is

$$z = \frac{1}{3} (\gamma_i^3 - 1) \frac{Mc^2}{E_0 (1 - \eta_e)^2} \text{ cm}. \quad (\text{II.9})$$

The axial acceleration increases the ion energy to γ_i ,

$$\gamma_i = (1 - \eta_e) \frac{r_1}{r_0}, \quad (\text{II.10})$$

where r_0 , r_1 are the major ring radii before and after the adiabatic expansion, respectively, E_0 is the electrostatic field of the ring before the expansion ($r = r_0$), and

$$\eta_e = \frac{N_i (\gamma_1 - 1) Mc^2}{N_e (\gamma_e - 1) mc^2} \quad (\text{II.11})$$

is the acceleration efficiency; i.e., the ratio of the total kinetic energy imparted to the ions to the total kinetic energy of the ring electrons.

For acceleration at low energy, say 1 GeV, it is possible to overload the ring with ions (for example, 60% neutralization). Then for a fivefold expansion ($r_1/r_0 = 5$), for example, we would have $\gamma_i = 2$. For $\gamma_e = 2000$ this would give $\eta_e = 0.6$, and at $E_0 = 19.2 \text{ MeV/cm}$ we would calculate an adiabatic expansion length of

$$z = 235 \text{ cm}.$$

When it is desired to achieve very high energy, $\eta_e \ll 0.1$; hence the correction $(1 - \eta_e)^2$ in equation (II.9) will be omitted in the calculation in the next section.

III. Acceleration of the Ions to Very High Energy

Following the phase of azimuthal acceleration of the ions and equalization of the azimuthal mechanical momentum of the ions and electrons it is desirable to start the ion acceleration at the

smallest practically possible radius of the electron ring. Therefore, before the acceleration of the ions the ring is compressed again to a field of 100,000 G. The ring and particle parameters at this phase are:

$$\begin{aligned} r &= 27 \text{ cm}, & p_{\theta e} &= 800 \text{ MeV/c}, \\ a &= 0.1 \text{ cm}, & \gamma_i &= 1.3, \\ I &= 36,700 \text{ A}, & p_{\theta i} &= 800 \text{ MeV/c}, \\ E &= 22 \text{ MeV/cm}, & p_{ze} = p_{zi} &= 0, \\ \zeta &= 0.74, & N_e &= 10^{15}, \\ \gamma_e &= 1600, & N_i &= 10^{11}. \end{aligned}$$

The length of the accelerator from the point of injection of the electrons until the above parameters are achieved is approximately 200 m. The ion acceleration is again a sequence of "rolling" in a decreasing magnetic field at constant ring radius and "coasting" in an adiabatic compression* but without a complete stop.

The length of the axial acceleration stage at constant radius is

$$z_j = \frac{2Mc^2}{3E_j \gamma_j^{1/2}} (\gamma_{j+1}^{3/2} - \gamma_j^{3/2}) \text{ cm} \quad (\text{III.1})$$

where γ_j , E_j are respectively the ion energy and the electrostatic field of the ring at the beginning of the j th acceleration stage, and γ_{j+1} is the ion energy at the end of the acceleration stage.

The length of the following adiabatic compression is

$$z_c = \frac{Mc^2}{3E_c \gamma_c^2} \gamma_{j+1}^3 - \gamma_c^3 \text{ cm} \quad (\text{III.2})$$

where γ_c , E_c are the ion energy and electrostatic field of the ring at the end of the compression stage and γ_{j+1} is the ion energy at the beginning of the adiabatic compression. Thus in each cycle of "roll" and "coast" the net energy gain of the ions is $\gamma_c - \gamma_i$. As an illustrative example the ring and particle parameters are listed in Table 1 for ion acceleration up to 1000 GeV. The symbol cr means acceleration at constant radius and ac the following phase of adiabatic compression.

The lengths indicated in Table 1 are calculated assuming that the ions are sitting just at the minor radius of the ring. Since this is not possible a reduction of the fields by 20% resulting in an increase of the total length to 670 m is necessary. Thus the overall length of the accelerator is approximately 870 m, when we add the 200 m of initial length mentioned previously. It should be noted, however, that in the last 530 m the accelerator is a tube less than 10 cm in diameter

* In an informal meeting on electron ring accelerators at Novosibirsk, Siberia, last summer, Harold Furth suggested the possibility of compressing the ring during the ion acceleration. To my knowledge no further development of this idea has been reported by Furth.

Table 1. Sample parameters for accelerating ions to 1000 GeV by subjecting the electron ring to alternating stages of acceleration at constant radius (cr) and adiabatic compression (ac).

B_z (G)	r (cm)	a (cm)	Mode	E (MeV/cm)	γ_i (rmu)	z (m)
100,000	27	0.1	cr	22	1.3	25.3
5,000	27	0.45	ac	4.9	200	12.4
36,500	10	0.17	cr	34.4	7.4	46.3
3,380	10	0.56	ac	10.4	80	26.0
37,000	3	0.19	cr	100	27	426
760	3	1.4		10.5	1000	
Total length						536

and the fields are in the few-thousand-gauss region. As the ion energy γ_i approaches γ_e the usual formula for $\gamma_i = \beta_{\theta e}^{-1}$ is not applicable anymore. The correct expression is

$$\gamma_i = \left[\left(\frac{B_f}{B_i} \right)^2 + \frac{1}{\gamma_e^2} \right]^{-1/2} \quad (\text{III.3})$$

where B_f is the final value of the axial magnetic field,

$$B_i = B_0 \frac{r_0}{r_f}, \quad (\text{III.4})$$

B_0 is the axial magnetic field at the beginning of the axial ion acceleration, and r_0 and r_f are the ring radii at the beginning and the end of acceleration. The condition

$$N_i > \frac{\gamma_i^2}{\gamma_e^2} N_e$$

is violated when $\gamma_i > 16$. At higher energies the ions are radially focused by the electric field of the ring reduced by γ_i^2 . The electrons are on their own and they are defocused by a field E/γ_e^2 . This field, however, is extremely small in comparison to the weak focusing provided by the external magnetic field, since $\gamma_e > 1000$. The electron ring is also unstable because the image charges exceed the image currents so that a net defocusing field E_d exists,

$$E_d = \frac{E}{\gamma_e^2} \frac{\xi}{R}, \quad (\text{III.5})$$

where ξ is a displacement of the ring axis from the axis of symmetry of the vacuum chamber, and R is the radius of the inner cylinder confining the internal coils. The highest field of the ring is 100 MeV/cm. Assuming $R = 2$ cm, $\xi = 0.1$ cm, and $\gamma_e = 1600$ we find that the defocusing field $E_d \approx 2$ V/cm. This is a very small field and it

can be easily compensated by the weak focusing external field.

It is of interest to find the length required for an e-folding of the displacement in the absence of external focusing. The equation of motion is

$$\ddot{\xi} = \left(\frac{eE}{mR} \right) \frac{\xi}{\gamma^3}. \quad (\text{III.6})$$

Then

$$\omega = \left(\frac{eE}{\gamma m R} \right)^{1/2} \cdot \gamma^{-1}.$$

The e-folding length $L = c/\omega$, which is

$$L = R \left(\frac{\gamma m c^2}{e E R} \right)^{1/2} \cdot \gamma. \quad (\text{III.7})$$

For $\gamma m c^2/e = 800$ MeV, $ER = 200$ MeV, $R = 2$ cm, and $\gamma = 1600$ we have

$$L = 6400 \text{ cm.}$$

Toward the end of the acceleration where $E \approx 10$ MeV/cm the exponentiation length is approximately 200 m. Thus during the acceleration from 16 to 1000 GeV even in the absence of external focusing there would have been only a few e-foldings of the axial displacement of the ring. Since displacement errors are changing at random along the accelerator at distances smaller than 60 m the effect would have been negligible. The above calculation was done to demonstrate that extremely relativistic particles ($\gamma > 1000$) will move very little in the radial direction even in the absence of external focusing.

Finally the energy spread required to quench the negative mass instability is calculated²:

$$\frac{\Delta E}{\gamma m c^2} > \left(\frac{2g}{\pi} \frac{r_e}{r} N_e \right)^{1/2} \cdot \gamma_e^{-1.5} \quad (\text{III.8})$$

where r_e is 2.8×10^{-13} , r is the ring radius, and $N_e \approx 10^{15}$ is the number of electrons in the ring. The geometric factor $2g/\pi$ is assumed of the order of unity.

At the initial radius of the ring ($r = 1000$ cm) the above condition becomes

$$\frac{\Delta E}{E} = \frac{0.53}{\gamma_e^{1.5}}. \quad (\text{III.8a})$$

Since $\gamma_e = 1600$ the required energy or momentum spread $\delta p \approx 0.01$ MeV/c.

The beam quality of the ion beam is extremely good. At the time of the trapping of the ions the transverse momentum is of the order of 50 MeV/c. As the radial amplitude increases during the acceleration about 15-fold the radial momentum is decreased to a few MeV/c while the total momentum is 10^6 MeV/c. Since the extracted beam is approximately 4 cm in diameter the final ion beam quality is only of the order of 10π microradian-cm. The only drawback of this beam is that its duration is of the order of 10^{-10} s. At operation of 60 pps the total yield of 1000-GeV protons is $1 \mu\text{A}$.

On the technological aspects of how to float the internal coils during the ion acceleration stages, it appears at this time that the best method is to hold them floating with ac magnetic fields. The phasing of these fields with the time of electron injection should be timed so that all the supporting magnetic fields vanish during the passage of the electron ring. An alternative resulting in substantial reduction of the length of the internal coils is to use adiabatic expansion above an ion energy of 250 GeV. This mode of operation would increase the length of the accelerator to about 1500 m while the length of the internal coils would be reduced to approximately 250 m.

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